

用颗粒相双尺度二阶矩湍流模型 模拟突扩两相流动

曾卓雄^{1,2}, 周力行¹, 张 健¹

(1. 清华大学工程力学系, 100084, 北京; 2. 南昌航空工业学院机械系, 330034, 南昌)

摘要: 基于将颗粒脉动分成湍流引起的大尺度脉动和颗粒间碰撞产生的小尺度脉动的概念, 应用单相流动多尺度湍流模型方法, 从双流体模型出发, 推导和封闭了颗粒的双尺度脉动雷诺应力方程、大尺度脉动能量传递率方程、小尺度脉动耗散率方程和两相脉动关联方程, 用统一的湍流模型方法构造了包含颗粒两类脉动的新的颗粒双尺度二阶矩两相湍流模型. 用该模型对突扩管内的两相流动进行了数值模拟, 并和单尺度的二阶矩两相湍流模型进行了比较, 计算结果和实验数据吻合较好, 比单尺度的二阶矩两相湍流模型的计算结果有所改进.

关键词: 两相流动; 二阶矩模型; 双尺度脉动

中图分类号: O359 **文献标识码:** A **文章编号:** 0253-987X(2006)01-0097-04

Simulation to Sudden-Expansion Two-Phase Flows with Two-Scale Second-Order Moment Particle Turbulence Model

Zeng Zhuoxiong^{1,2}, Zhou Lixing¹, Zhang Jian¹

(1. Department of Engineering Mechanics, Tsinghua University, Beijing 100084, China; 2. Department of Mechanical Engineering, Nanchang Institute of Aeronautical Technology, Nanchang 330034, China)

Abstract: A two-scale second-order moment particle turbulence model accounting for inter-particle collision is developed based on the concept that the particle fluctuation can be divided into large-scale one due to turbulence and small-scale one due to collision with a unified treatment. Adopting the multiple scale of single-fluid method and two-fluid model, the particle Reynolds stress equation for two-scale fluctuation, the energy transfer rate equation for large-scale fluctuation, the turbulent kinetic energy dissipation rate equation for small-scale fluctuation, and the gas-particle velocity correlation equation for two-scale fluctuation are all derived and closed. The presented model is employed to simulate gas-particle flows in a sudden-expansion chamber, and the simulated results coincide well with the experimental ones. It is also demonstrated that the presented two-scale model is with higher calculating accuracy than the single-scale model.

Keywords: gas-particle flow; second-order moment model; two-scale fluctuation

两相湍流及其相互作用是当前国际上研究的热点之一, 为此出现了众多的两相湍流模型^[1-6]. 另一方面, 大多数单相流动湍流模型都基于单个时间尺度及长度尺度, 然而实际的湍流脉动包含了很宽的尺度范围. 文献[7]根据湍流谱分布的特点, 提出了

双时间尺度的湍流模型, 文献[8]则进一步得到了双尺度 $k-\epsilon$ 模型方程. 文献[9]根据流体湍流中大涡旋和小涡旋性质上的不同以及大涡旋中由外来扰动和局部产生的大涡旋的差异, 提出了一个把 3 种湍流涡旋分开考虑的含 3 种速度脉动量的多尺度雷诺应

力方程,但是没有实际加以应用.文献[10]将时间常数和模型系数引入到多尺度的雷诺应力输运方程中,得到了多尺度的能量传递率的输运方程.

在两相流动的双流体模型方面,国内外尚没有进行颗粒双尺度湍流模型的研究.本文基于颗粒脉动包含大尺度的湍流脉动和颗粒间碰撞引起的小尺度脉动的概念(认为颗粒间的碰撞是近距离作用,它直接引起小尺度脉动(湍流耗散尺度),间接影响大尺度脉动(湍流基本尺度),大尺度脉动和小尺度脉动通过能量传递率相互作用),应用单相流动多尺度湍流模型方法,从双流体模型出发,推导和封闭了颗粒的双尺度雷诺应力方程、大尺度脉动能量传递率方程、小尺度脉动耗散率方程和两相脉动关联方程,提出了一种新的颗粒双尺度二阶矩两相湍流模型,用统一的湍流模型方法来构造包含颗粒两类脉动的模型(气相仍然采用单尺度的二阶矩模型),并且用该模型对突扩管内的两相流动进行了数值模拟,同时和单尺度的统一二阶矩(USM)两相湍流模型进行了比较.

1 双尺度二阶矩两相湍流模型

如果只考虑重力和阻力,对无反应的等温湍流气粒两相流动从两相瞬时方程组出发,可以导出视密度加权平均的统一二阶矩两相湍流模型的基本方程组.对气体仍然采用单尺度湍流模型,见文献[11].下文中用大写字母来表示平均量,小写字母表示脉动量, α_p 为颗粒体积浓度, β 为阻力系数, U 为平均速度, u 、 v 分别为轴向、径向脉动速度, uu 、 vv 为雷诺应力或脉动速度关联, e 为碰撞恢复系数, d 为颗粒直径, k 为湍动能, ε 为湍动能耗散率;下角标中1为大尺度脉动,2为小尺度脉动, p 为颗粒, f 为气体.将颗粒的某一物理量的瞬时值 Φ_p 分解成3部分——质量加权平均值 Φ_p ,大尺度脉动值 ϕ_p ,小尺度脉动值 ϕ_p ,从而有: $\Phi_p = \Phi_p + \phi_p + \phi_p = \Phi_p + \phi_p$.如果略去速度平均值的平均符号,可以获得如下的颗粒连续方程和动量方程:

颗粒连续方程

$$\frac{\partial(\bar{\alpha}_p \rho_p)}{\partial t} + \frac{\partial(\bar{\alpha}_p \rho_p U_{pk})}{\partial x_k} = 0 \quad (1)$$

颗粒动量方程

$$\begin{aligned} \frac{\partial}{\partial t}(\bar{\alpha}_p \rho_p U_{pi}) + \frac{\partial}{\partial x_k}(\bar{\alpha}_p \rho_p U_{pi} U_{pk}) = & -\bar{\alpha}_p \frac{\partial P_i}{\partial x_i} - \\ & \frac{\partial}{\partial x_k}(\bar{\alpha}_p \rho_p \overline{u_{1pi} u_{1pk}}) - \frac{\partial}{\partial x_k}(\bar{\alpha}_p \rho_p \overline{u_{2pi} u_{2pk}}) - \end{aligned}$$

$$G \frac{\partial \bar{\alpha}_p}{\partial x_k} + \frac{\partial \bar{\alpha}_p \tau_{pjk}}{\partial x_k} + \beta(U_{fi} - U_{pi}) + \bar{\alpha}_p \rho_p g_i \quad (2)$$

颗粒应力模数 G 的表达式及颗粒的黏性系数可参考文献[12].

认为颗粒的大尺度脉动能量来自平均运动,小尺度脉动的产生项来自大尺度脉动的能量传递以及颗粒碰撞的作用,可以获得颗粒的双尺度雷诺应力输运方程.

颗粒大尺度脉动雷诺应力输运方程为

$$\begin{aligned} \frac{\partial(\bar{\alpha}_p \rho_p \overline{u_{1pi} u_{1pj}})}{\partial t} + \frac{\partial(\bar{\alpha}_p \rho_p \overline{U_{pk} u_{1pi} u_{1pj}})}{\partial x_k} = & D_{1p,ij} + \\ & P_{p,ij} + \Pi_{p,ij} - \bar{\alpha}_p \rho_p T_{p,ij} + G_{1p,pf,ij} \quad (3) \end{aligned}$$

式中右端各项依次为扩散项、产生项、压力应变项、能量传递率和两相湍流相互作用项,其中

$$\begin{aligned} D_{1p,ij} = & \frac{\partial}{\partial x_k} \left[C_p \bar{\alpha}_p \rho_p \frac{k_{1p}}{T_{p,kk}} \overline{u_{1pk} u_{1pj}} \frac{\partial \overline{u_{1pi} u_{1pj}}}{\partial x_i} \right] \\ P_{p,ij} = & -\rho_p \bar{\alpha}_p \left[\overline{(u_{1pi} u_{1pk} + u_{2pi} u_{2pk})} \frac{\partial U_{pi}}{\partial x_k} + \right. \\ & \left. \overline{(u_{1pj} u_{1pk} + u_{2pj} u_{2pk})} \frac{\partial U_{pj}}{\partial x_k} \right] \end{aligned}$$

$$\begin{aligned} \Pi_{p,ij} = & -C_{1p1} \frac{T_{p,kk}}{k_{1p}} \bar{\alpha}_p \rho_p \left[\overline{u_{1pi} u_{1pj}} - \frac{2}{3} k_{1p} \delta_{ij} \right] - \\ & C_{1p2} \left[P_{p,ij} - \frac{2}{3} P_p \delta_{ij} \right] \end{aligned}$$

$$P_p = -\rho_p \bar{\alpha}_p \left(\overline{u_{1pi} u_{1pk}} + \overline{u_{2pi} u_{2pk}} \right) \frac{\partial U_{pi}}{\partial x_k}$$

$$G_{1p,pf,ij} = \beta \left[\overline{u_{1pj} u_{fi}} + \overline{u_{1pi} u_{fj}} - 2 \overline{u_{1pi} u_{1pj}} \right]$$

颗粒小尺度脉动雷诺应力输运方程为

$$\begin{aligned} \frac{\partial(\bar{\alpha}_p \rho_p \overline{u_{2pi} u_{2pj}})}{\partial t} + \frac{\partial(\bar{\alpha}_p \rho_p \overline{U_{pk} u_{2pi} u_{2pj}})}{\partial x_k} = & D_{2p,ij} + \\ & \bar{\alpha}_p \rho_p T_{p,ij} - \varepsilon_{p,ij} + J_{p,ij} + G_{2p,pf,ij} \quad (4) \end{aligned}$$

式中右端各项依次为扩散项、来自大尺度脉动的能量传递率、耗散率、各方向湍流间的相互作用项(来自颗粒间的碰撞)和两相湍流相互作用项,分别为

$$\begin{aligned} D_{2p,ij} = & \frac{\partial}{\partial x_k} \left[C_p \bar{\alpha}_p \rho_p \frac{k_{2p}}{\varepsilon_p} \overline{u_{2pk} u_{2pj}} \frac{\partial \overline{u_{2pi} u_{2pj}}}{\partial x_i} \right] \\ \varepsilon_{p,ij} = & \frac{2}{3} \bar{\alpha}_p \rho_p \varepsilon_p \delta_{ij} \end{aligned}$$

$$G_{2p,pf,ij} = \beta \left(\overline{u_{2pj} u_{fi}} + \overline{u_{2pi} u_{fj}} - 2 \overline{u_{2pi} u_{2pj}} \right)$$

$$J_{p,ij} = -\frac{2}{9} (1 - e^2) \frac{k_{2p} \bar{\alpha}_p \rho_p}{\tau_c} \delta_{ij} -$$

$$\frac{2 \bar{\alpha}_p \rho_p}{\tau_{c1}} \left[\overline{u_{2pi} u_{2pj}} - \frac{2}{3} k_{2p} \delta_{ij} \right]$$

其中

$$\tau_c = \left(\frac{2\pi}{3k_{2p}} \right)^{1/2} \frac{d_p}{16 \bar{\alpha}_p}$$

$$\tau_{c1} = \left(\frac{2\pi}{3k_{2p}} \right)^{1/2} \frac{5d_p}{8(1+e)(3-e)\bar{\alpha}_p}$$

大尺度脉动能量传递率是最难构造准确模型的项,本文采用的能量传递率方程封闭的基本思想是:能量传递率的生成、扩散、耗散等项和大尺度脉动雷诺应力方程中的对应项有类似的机制和公式.为了反映大尺度脉动和小尺度脉动之间变化的分界波数,本文在大尺度脉动能量传递率方程中引入 $C_{p4}\bar{\alpha}_p\rho_p P_{p,ij}/k_{1p}$,在小尺度脉动耗散率方程中引入 $C_{p4}\bar{\alpha}_p\rho_p T_{p,kk}/k_{2p}$.

大尺度脉动能量传递率方程为

$$\frac{\partial(\bar{\alpha}_p\rho_p T_{p,ij})}{\partial t} + \frac{\partial(\bar{\alpha}_p\rho_p U_{pk} T_{p,ij})}{\partial x_k} = \frac{\partial}{\partial x_k} \left[\bar{\alpha}_p\rho_p C_p^d \frac{k_{1p}}{T_{p,kk}} \overline{u_{1pk} u_{1pl}} \frac{\partial T_{p,ij}}{\partial x_l} \right] + \frac{T_{p,kk}}{k_{1p}} (C_{p1} P_{p,ij} - C_{p2} \bar{\alpha}_p\rho_p T_{p,ij} + C_{p3} G_{1p,ij}) + C_{p4} \bar{\alpha}_p\rho_p \frac{P_{p,ij}^2}{k_{1p}} \quad (5)$$

小尺度脉动耗散率方程为

$$\frac{\partial(\bar{\alpha}_p\rho_p \varepsilon_p)}{\partial t} + \frac{\partial(\bar{\alpha}_p\rho_p U_{pk} \varepsilon_p)}{\partial x_k} = \frac{\partial}{\partial x_k} \left[\bar{\alpha}_p\rho_p C_p^d \frac{k_{2p}}{\varepsilon_p} \overline{u_{2pk} u_{2pl}} \frac{\partial \varepsilon_p}{\partial x_l} \right] + \frac{\varepsilon_p}{k_{2p}} (C_{p1} \bar{\alpha}_p\rho_p T_{p,kk} - C_{p2} \bar{\alpha}_p\rho_p \varepsilon_p + C_{p3} G_{2p,pf}) + C_{p4} \frac{T_{p,kk}^2}{k_{2p}} \bar{\alpha}_p\rho_p \quad (6)$$

式中

$$G_{2p,pf} = 2\beta (\overline{u_{2pi} u_{fi}} - \overline{u_{2pi} u_{2pi}})$$

在雷诺应力方程中,出现了颗粒大尺度脉动和气体速度脉动的关联项以及颗粒小尺度脉动和气体速度脉动的关联项,因此需要另外建立方程来进行封闭.由气体和颗粒的瞬时动量方程可推导出这两个关联方程.颗粒大尺度脉动和气体速度脉动关联的方程为

$$\frac{\partial \overline{u_{1pi} u_{fi}}}{\partial t} + (U_{fk} + U_{pk}) \frac{\partial \overline{u_{1pi} u_{fi}}}{\partial x_k} = D_{1pf,ij} + P_{1pf,ij} + \Pi_{1pf,ij} - \varepsilon_{1pf,ij} + G_{1pf,ij} \quad (7)$$

式中右端各项依次为扩散项、产生项、压力应变项、耗散项和两相相互作用项,其中

$$\begin{aligned} D_{1pf,ij} &= \frac{\partial}{\partial x_k} \left[C_{1pf3} \left(\frac{k_{1p}}{T_{p,kk}} \overline{u_{1pk} u_{1pl}} + \frac{k_f}{\varepsilon_f} \overline{u_{fk} u_{fl}} \right) \frac{\partial \overline{u_{1pi} u_{fi}}}{\partial x_l} \right] \\ P_{1pf,ij} &= - \overline{u_{1pi} u_{fk}} \frac{\partial U_{fi}}{\partial x_k} - \overline{u_{fi} u_{1pk}} \frac{\partial U_{pi}}{\partial x_k} \\ P_{1pf} &= \frac{1}{2} \left(- \overline{u_{1pi} u_{fk}} \frac{\partial U_{fi}}{\partial x_k} - \overline{u_{fi} u_{1pk}} \frac{\partial U_{pi}}{\partial x_k} \right) \\ \Pi_{1pf,ij} &= - \frac{C_{1pf1}}{\tau_p} \left(\overline{u_{1pi} u_{fj}} - \frac{2}{3} k_{1pf} \delta_{ij} \right) - \end{aligned}$$

$$\begin{aligned} C_{1pf,2} &= \left(P_{1pf,ij} - \frac{2}{3} P_{1pf} \delta_{ij} \right) \\ G_{1pf,ij} &= \frac{\beta}{\bar{\alpha}_f \rho_f \bar{\alpha}_p \rho_p} [\bar{\alpha}_p \rho_p \overline{u_{1pi} u_{pj}} + \bar{\alpha}_f \rho_f \overline{u_{fi} u_{fj}} - (\bar{\alpha}_p \rho_p + \bar{\alpha}_f \rho_f) \overline{u_{1pi} u_{fj}}] \\ \varepsilon_{1pf,ij} &= c_{p1f} \frac{\varepsilon_i}{k_f} \overline{u_{1pi} u_{fi}} \end{aligned}$$

颗粒小尺度脉动和气体速度脉动关联的方程为

$$\frac{\partial \overline{u_{2pi} u_{fi}}}{\partial t} + (U_{fk} + U_{pk}) \frac{\partial \overline{u_{2pi} u_{fi}}}{\partial x_k} =$$

$$D_{2pf,ij} + P_{2pf,ij} + \Pi_{2pf,ij} - \varepsilon_{2pf,ij} + G_{2pf,ij} \quad (8)$$

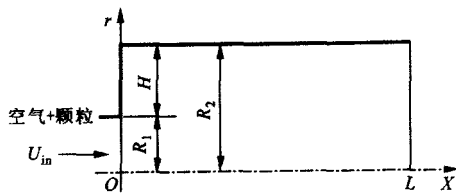
式中右端各项依次为扩散项、产生项、压力应变项、耗散项和两相相互作用项,其中

$$\begin{aligned} D_{2pf,ij} &= \frac{\partial}{\partial x_k} \left[C_{2pf3} \left(\frac{k_{2p}}{\varepsilon_{2p}} \overline{u_{2pk} u_{2pl}} + \frac{k_f}{\varepsilon_f} \overline{u_{fk} u_{fl}} \right) \frac{\partial \overline{u_{2pi} u_{fi}}}{\partial x_l} \right] \\ P_{2pf,ij} &= - \overline{u_{2pi} u_{fk}} \frac{\partial U_{fi}}{\partial x_k} - \overline{u_{fi} u_{2pk}} \frac{\partial U_{pi}}{\partial x_k} \\ P_{2pf} &= \frac{1}{2} \left(- \overline{u_{2pi} u_{fk}} \frac{\partial U_{fi}}{\partial x_k} - \overline{u_{fi} u_{2pk}} \frac{\partial U_{pi}}{\partial x_k} \right) \\ \Pi_{2pf,ij} &= - \frac{C_{2pf1}}{\tau_p} \left(\overline{u_{2pi} u_{fj}} - \frac{2}{3} k_{2pf} \delta_{ij} \right) - C_{2pf,2} \left(P_{2pf,ij} - \frac{2}{3} P_{2pf} \delta_{ij} \right) \\ G_{2pf,ij} &= \frac{\beta}{\bar{\alpha}_f \rho_f \bar{\alpha}_p \rho_p} [\bar{\alpha}_p \rho_p \overline{u_{2pi} u_{pj}} + \bar{\alpha}_f \rho_f \overline{u_{fi} u_{fj}} - (\bar{\alpha}_p \rho_p + \bar{\alpha}_f \rho_f) \overline{u_{2pi} u_{fj}}] \\ \varepsilon_{2pf,ij} &= c_{p2f} \frac{\varepsilon_i}{k_f} \overline{u_{2pi} u_{fi}} \end{aligned}$$

式(1)到式(8)加上气体的连续、动量和雷诺应力方程,构成了考虑颗粒间碰撞的双尺度二阶矩两相湍流模型.本模型的特点是用统一的理论同时考虑颗粒的大尺度湍流脉动和颗粒间碰撞引起的小尺度脉动,从数学上看,本模型的方程数目增多,使得计算所需时间加长,但是从物理的角度上看,本模型的概念更为清楚,能够对方程各自的特点进行合理的假设和简化.

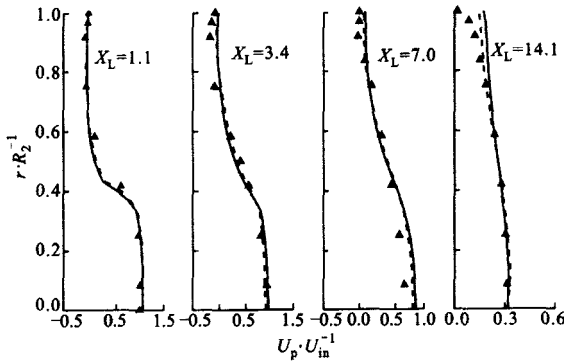
2 轴对称突扩气粒两相流动的数值模拟

所模拟的突扩室的几何形状和尺寸如图 1 所示,实验结果可参考文献[13].模拟中采用节点数 61×201 的均匀交错网格.用本模型和单尺度 USM 两相湍流模型对突扩两相流动进行了数值模拟.模拟结果如图 2 到图 6 所示,图中 $X_L = X/H$.由图 2 可以看出,两种不同的两相湍流模型计算出来的颗粒轴向平均速度和实验测量值符合得都很好,在



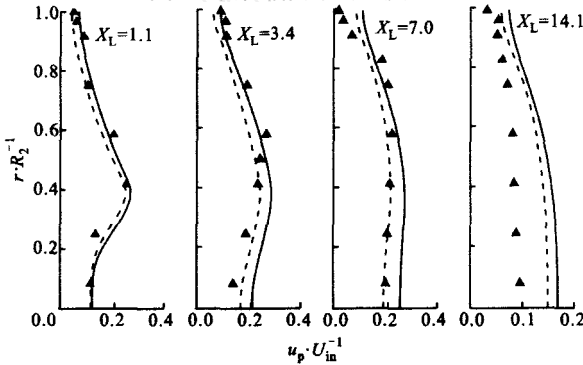
$R_1 = 0.025 \text{ m}$; $H = 0.035 \text{ m}$; $R_2 = 0.06 \text{ m}$; $L = 1 \text{ m}$

图 1 突扩管道示意图



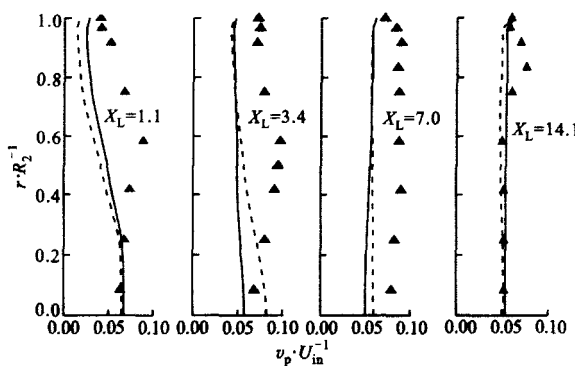
▲实验; — USM 模型; — 本模型

图 2 颗粒轴向平均速度



▲实验; — USM 模型; — 本模型

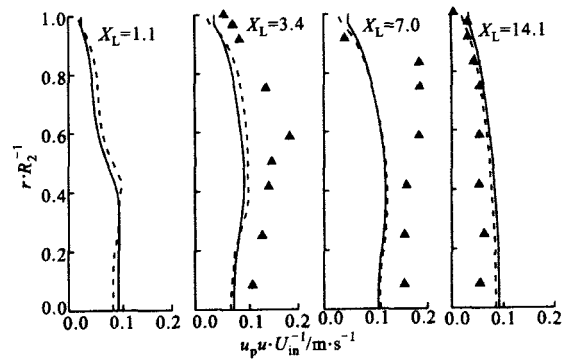
图 3 颗粒轴向脉动速度



▲实验; — USM 模型; — 本模型

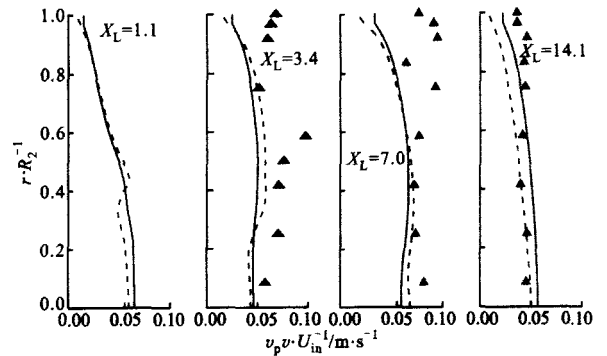
图 4 颗粒径向脉动速度

$X = 14.1 H$ 的壁面处,本模型的预报值和实验值符合得更好.从图 3 和图 4 可以看出,本模型对颗粒轴向脉动速度的预报结果比 USM 模型有所改进,但



▲实验; — USM 模型; — 本模型

图 5 两相轴向脉动速度关联



▲实验; — USM 模型; — 本模型

图 6 两相径向脉动速度关联

对颗粒径向脉动速度的预报结果,除 $X = 7 H$ 的位置外,则不如 USM 模型.图 5 和图 6 分别给出了两相脉动速度关联的轴向和径向分量,显示本模型对轴向分量的预报比 USM 模型略好.在 $X = 7 H$ 处,两种模型得出的结果很接近,然而在径向分量的预报中,两种模型的预报结果在壁面处都比实验值小.本模型在大部分区域的预报结果优于 USM 模型,但是在另外一小部分区域的预报结果则比 USM 模型差.从总体上看,本模型比单尺度 USM 模型有一定程度的改进.

3 结 论

基于将颗粒脉动分成湍流引起的大尺度脉动和颗粒间碰撞产生的小尺度脉动的概念,建立了一种新的颗粒双尺度二阶矩两相湍流模型,对突扩管道内两相流动的模拟表明,本模型比单尺度的二阶矩两相湍流模型有一定程度的改进.

今后需要开展进一步的研究,用 DNS 或 LES 的结果来检验和改进本模型的封闭.

(下转第 110 页)

参考文献:

- [1] 林宗虎. 管路内气液两相流特性及其工程应用[M]. 西安: 西安交通大学出版社, 1992.
- [2] Azzopardi B J, Whalley P B. The effect of flow pattern on two-phase flow in a T-junction[J]. Int J Multiphase Flow, 1982, 8(5): 491-507.
- [3] Shoham O, Brill J P. Two-phase flow splitting in a tee junction experiment and modelling[J]. Chem Engng Sci, 1987, 42(11): 2667-2676.
- [4] Hwang S T, Soliman H M. Phase separation in dividing two-phase flow[J]. Int J Multiphase Flow, 1988, 14(4): 439-458.
- [5] Hart J, Hamersma P J. A model for predicting liquid route preference during gas-liquid flow through horizontal branched pipelines[J]. Chem Engng Sci, 1991, 46(7): 1609-1662.
- [6] Gomez L E, Shoham O. A unified mechanistic model for steady-state two-phase flow in wellbores and pipelines[A]. SPE Annular Technical Conference and Exhibition. Houston, USA, 1999.

(编辑 王焕雪)

(上接第100页)

参考文献:

- [1] Zhou L X, Huang X Q. Prediction of confined gas-particle jets by an energy equation model of particle turbulence[J]. Science in China, 1990, 33(1): 53-59.
- [2] Zhou L X, Liao C M, Chen T. A unified second-order moment two-phase turbulence model for simulating gas-particle flows[A]. Crowe C T, Johnson R. Numerical Methods in Multiphase Flows, ASME-FED[C]. Washington: Washington State University, 1994. 307-313.
- [3] Cheng Y, Guo Y C, Wei F, et al. Modeling the hydrodynamics of downer reactors based on kinetic theory[J]. Chem Eng Sci, 1999, 54(13-14): 2019-2027.
- [4] Zheng Y, Wan X T, Qian Z, et al. Numerical simulation of the gas-particle turbulent flow in riser reactor based on $k\text{-}\varepsilon\text{-}k_p\text{-}\varepsilon_p\text{-}\Theta$ two-fluid model[J]. Chem Eng Sci, 2001, 56(24): 6813-6822.
- [5] 曾卓雄, 胡春波, 姜培正, 等. 密相、湍流、可压、气固两相流理论及在管道中的应用[J]. 空气动力学学报, 2001, 19(1): 109-118.
- [6] Yu Y, Zhou L X. A second-order moment two-phase turbulence model for dense gas-particle flows[A]. Matsumoto Y, Hishida K. Proc 5th Int Conf on Multiphase Flow[C/CD]. Tokyo: Tokyo Institute of Technology, 2004. Paper 161.
- [7] Hanjalic K, Launder B E, Schiestel R. Multiple-time-scale concepts in turbulent transport modeling[A]. Bradbur L J S. Turbulent Shear Flows: 2[C]. New York: Springer-Verlag, 1980. 36-49.
- [8] Kim S W, Chen C P. A multiple-time-scale turbulence model based on variable partitioning of the time turbulent kinetic energy spectrum[J]. Numerical Heat Transfer: Part B, 1989, 16(2): 193-211.
- [9] 蔡树棠, 刘宇陆. 湍流理论[M]. 上海: 上海交通大学出版社, 1993.
- [10] Yonemura S, Yamamoto M. Proposal of a multiple-time-scale Reynolds stress mode for turbulent boundary layers[A]. Daiguji H, Miyake Y. Proc Int Symp on Mathematical Modeling of Turbulent Flows[C]. Tokyo: The University of Tokyo, 1995. 381-386.
- [11] 周力行. 多相湍流反应流体力学[M]. 北京: 国防工业出版社, 2002.
- [12] Tsuo Y P, Gidaspow D. Computation of flow patterns in circulating fluidized beds[J]. AIChE J, 1990, 36(6): 885-896.
- [13] Xu Y, Zhou L X. Experimental studies on two-phase fluctuation velocity correlation in sudden-expansion flows[A]. Stock D E. Proceedings of ASME-FEDSM '99[C/CD]. Washington: Washington State University, 1999. Paper FEDSM99-7909.

(编辑 葛赵青)